CHAPTER 13

SI System of Units

The SI System of Units (Le Système International d’Unités) is probably familiar to you as the “metric system.” This system is used worldwide except for the United States, Liberia, and Myanmar (Burma). It is the system favored by science, so you undoubtedly have been exposed to it in your science classes.

13.1 HISTORICAL BACKGROUND

The need for units of measure was evident as soon as human commerce began. If two farmers were to trade grain for a goat, they needed to quantify the amount of grain and the weight of the goat. In early commerce, the units of measure were based on commonly available items. For example, the bushel basket used to transport the grain became the unit of measure. (In Britain, the bushel was eventually standardized to be eight imperial gallons.) The weight of the goat could be measured by placing the animal on a scale and determining the number of stones required to counterbalance the animal. (In Britain, the stone was eventually defined to be 14 pounds.) The unit of length, based on a man’s foot, has been used in Britain for over a thousand years. It quickly became evident that units of measure had to be subdivided. Many ancient measuring systems were based upon fractions of the base unit, such as halves, thirds, and quarters. Thus, the unit was subdivided into a number of segments that is easily divided into fractions. For example, the foot is divided into 12 inches, which may be evenly divided by 2, 3, 4, and 6 with no remainder.

For units of measure to be useful, they must be standardized so that business transactions are unambiguous. Thus, it fell upon governments to establish official units of measure. For example, the Egyptian Royal Cubit was equivalent to the length from the Pharaoh’s elbow to the farthest fingertip of his extended hand (20.62 inches). A block of granite was fashioned to this length to become a standard. (After all, the Pharaoh was much too busy to help carpenters measure the lengths of boards.) This standard was further divided into finger widths, palms, hands, remens (20 finger widths), and a small cubit (18 inches) equal to six palms (3 inches). The small cubit was used widely in construction and was fashioned into wood or granite copies that were regularly checked against the standard. We continue to use the Pharaoh’s system of measure—the height of horses is often measured in hands, which are now defined to be exactly four inches.
In the 16th century, decimal systems were conceived in which the units of measure were divided into 10 parts, 100 parts, 1000 parts, and so on, rather than fractional divisions. This allowed for more accurate and convenient subdivisions; however, as there were no standards, confusion abounded. In 1790, the French National Assembly requested that the French Academy of Sciences establish a system of units that could be adopted the world over. It used the meter as the unit of length and the gram as the unit of mass. This system was legalized in the United States in 1866.

In 1870, an international meeting was held in Paris in which 15 nations were represented. This led to the establishment of the International Bureau of Weights and Measures near Paris. They agreed to hold the General Conference on Weights and Measures at least every six years to decide upon issues relating to units of measure. The National Institute of Standards and Technology (NIST), formerly the National Bureau of Standards (NBS), represents the United States at these meetings.

Any measuring system must establish base units from which all other units are derived. (For example, volume is derived from the base unit of length.) In 1881, time was added as a third base unit to establish the centimeter-gram-second (CGS) system. Outside of the laboratory, this system is inconvenient, so about 1900, the meter-kilogram-second (MKS) was adopted. In 1935, electrical measurements based on the ampere were added. Thus there was a fourth base unit in the MKSA system. In 1954, base units for temperature (kelvin) and luminous intensity (candela) were adopted, bringing the total base units to six. In 1960, the measurement system was given the formal title, Le Système International d’Unités, which we abbreviate as SI. In 1971, the amount of substance (mole) was added as a base unit, bringing the total to seven.

### 13.2 DIMENSIONS AND UNITS

The distinction between a *dimension* and a *unit* is best understood by example. The *dimension* of length may be described by units of meters, feet, inches, cubits, and so forth. Thus, *dimension* is an abstract idea whereas *unit* is more specific. Table 13.1 shows common dimensions and the associated SI base units.

#### TABLE 13.1
Dimensions and SI base units

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Symbol</th>
<th>Unit</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>[L]</td>
<td>meter</td>
<td>m</td>
</tr>
<tr>
<td>Mass</td>
<td>[M]</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>Time</td>
<td>[T]</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>Electric current</td>
<td>[A]</td>
<td>ampere</td>
<td>A</td>
</tr>
<tr>
<td>Thermodynamic temperature</td>
<td>[θ]</td>
<td>kelvin</td>
<td>K</td>
</tr>
<tr>
<td>Luminous intensity</td>
<td>[I]</td>
<td>candela</td>
<td>cd</td>
</tr>
<tr>
<td>Amount of substance</td>
<td>[N]</td>
<td>mole</td>
<td>mol</td>
</tr>
</tbody>
</table>
13.3 SI UNITS

SI includes three types of units: supplementary, base, and derived.

13.3.1 SI Supplementary Units

The SI supplementary units were added in 1960. They are mathematical definitions that are needed to define both base and derived units.

1. Plane Angle (radian). Figure 13.1 shows a circle in which two radii define a plane angle $\theta$. If the length of the swept circumference is equal to the circle radius, then the plane angle $\theta$ is equal to one radian (1 rad).

Born in Revolution

The erratic behavior and bankruptcy of Louis XVI led to the disintegration of French social order and culminated in the storming of the Bastille on July 14, 1789. This event initiated the destruction of the feudal system as peasants were emboldened to torch chateaux, burn feudal contracts, take land, and drive out the gentry. Political power shifted from the king to the French National Assembly.

In 1790, the French National Assembly requested that the French Academy of Sciences revise the French system of weights and measures, which, under the monarchy, were emboldened to torch chateaux, burn feudal contracts, take land, and drive out the gentry. They appointed a “blue-ribbon” panel headed by Jean Charles Borda (1733–1799). Invitations to sit on the committee were sent to both Britain and the United States, but these were declined.

The committee decided the measuring system should be base 10, although base 11 and 12 were considered. The unit of length, the meter, was to be one 10-millionth of a “quadrant of meridian,” that is, the distance from the north pole to the equator measured along a great circle passing through the poles. The unit of mass, the gram, was to be the mass of water, at its maximum density ($4^\circ$C), occupying a volume of $10^{-6}$ m$^3$.

Two French astronomer-geodesists were given the task of surveying the distance from Dunkirk, France, to a site near Barcelona, Spain, from which the quadrant of meridian could be calculated. It took them seven years to complete the measurements; their work was impeded because they were arrested for spying while surveying foreign countries. They were remarkably accurate, with an error of only 2 parts in 10,000.

In 1792, the newly elected National Convention proclaimed France to be a republic. They severed all ties with the traditional Gregorian calendar and established this as Year 1 of the Republic of France. To create a rational calendar, they established a new commission, which devised a 12-month calendar with each month exactly 30 days. To complete the 365-1/4 days in a year, each year had a 5-day festival, except during leap year, which had a 6-day festival. Rather than the traditional 7-day week, the month was divided into three 10-day décades. Rather than naming each day after gods and goddesses, the days were numbered from one to ten. This calendar was employed for over 12 years, until Napoleon abandoned it in 1806.

The calendar commission also proposed a decimal system of time. Each day was divided into ten decidays (2.4 hours); smaller units were the milliday (86.4 seconds) and microday (0.0864 seconds). In 1793, the decimal time system was introduced, but was met with stiff resistance. Unlike the other weights and measures employed by the monarchy, the system for time measurement was rigorous, well structured, and universally followed. Changing all the clocks would be expensive. In addition, time is intimately connected with people’s everyday lives, whereas the units for length and mass are less interwoven. There was little incentive for change, so in 1795, the proposed decimal time system was “tabled” and has remained so ever since.

In 1798, European scientists were invited to France to continue improving the new “metric system.” Eventually, it became a measuring system that was adopted by nearly the entire world. Interestingly, the United States, which was invited to attend the very first meetings, has had the greatest difficulty adopting this measurement system.

Any plane angle $\theta$ is defined as the length of the swept circumference divided by the radius:

$$\theta = \frac{\text{Swept Circumference}}{\text{Radius}} = \frac{[\text{L}]}{[\text{L}]}$$  \hspace{1cm} (13-1)

Both swept circumference and radius have dimensions of length, so plane angles have dimensions of $[\text{L}/\text{L}]$. In SI, the unit for length is the meter (m), so the SI plane angle unit is m/m. These units cancel each other, so plane angles are commonly written without their associated units. Although not formally required, some people prefer to include the abbreviation $\text{rad}$ after the plane angle.

2. **Solid Angle (steradian).** Figure 13.2 shows a sphere in which four radii define a surface with area $A$. If $A = r^2$, then the solid angle $\beta$ is equal to one steradian (1 sr).

Any solid angle is defined as the swept area divided by the radius squared:

$$\beta = \frac{\text{Swept Area}}{(\text{Radius})^2} = \frac{[\text{L}^2]}{[\text{L}]^2}$$  \hspace{1cm} (13-2)

The swept area has dimensions of length squared. Radius has dimensions of length, but because the radius is squared, the denominator also has dimensions of length squared. In SI, the unit for length is the meter (m), so the SI plane angle unit is m$^2$/m$^2$. Again, these units cancel each other, so solid angles are commonly written without their associated units. Although not formally required, some people prefer to include the abbreviation $\text{sr}$ after the solid angle.

### 13.3.2 SI Base Units

The base units are defined as follows:

1. **Unit of Length (meter).** The meter was first defined in 1793 by dividing the “quadrant of meridian” (the length from the north pole to the equator measured along a great circle passing through the poles) into 10 million parts. After surveying the earth to determine the length of the quadrant of meridian, the meter was reproduced in three platinum bars and several iron bars. Due to surveying error, it was later found that the bar lengths did not correspond exactly to the original definition. Rather than change the bar lengths, the original definition was abandoned. Because the platinum bars are not easily transported and because they had to be stored at an exact temperature (i.e., the temperature of melting ice) to maintain a given length, they were abandoned as a standard in 1960. Currently, the meter is defined using the distance light traverses in a given length of time.

   *The meter is the length of the path traveled by light in vacuum during a time interval of $1/299792458$ of a second.*

2. **Unit of Mass (kilogram).** In 1799, the kilogram was defined as the mass of pure water at the temperature of its maximum density ($4^\circ\text{C}$) that occupies a cubic decimeter (0.001 m$^3$). It was later determined that the standard volume used to measure the water was actually 1.000028 cubic decimeters. This definition of the kilogram was abandoned in 1889.
The kilogram is defined by a cylindrical prototype composed of an alloy of platinum and 10% iridium maintained under vacuum conditions near Paris.

The kilogram is the only base unit that is not transportable. Copies are made that match the mass of the original by 1 part in $10^8$ or better. Unfortunately, metallurgy in the 19th century was not sophisticated, so impurities in the platinum-iridium cause detectable changes in the prototype mass of about 0.5 part per billion every year. Thus, the definition of the kilogram changes with time.

3. Unit of Time (second). The unit of time was originally defined as 1/86400 of the mean solar day. Because of irregularities in the earth’s rotation, the definition was changed to the “ephemeris second,” i.e., 1/31556925.9747 of the tropical year 1900. In 1967, this definition was replaced.

   The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium-133 atom.

This definition is based on the atomic clock. The best atomic clock (NIST-7) is precise to within about one second in three million years, or 1 part in $10^{14}$. Commercially available atomic clocks are precise to within 3 parts in $10^{12}$.

4. Unit of Electric Current (ampere). When electric current flows through a wire, a magnetic field surrounds the wire. The ampere was defined in 1948 based on the magnetic force of attraction between two parallel wires with electric current flowing.

   The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed one meter apart in a vacuum, would produce between these conductors a force equal to $2 \times 10^{-7}$ newton per meter of length.

This can be better understood by considering Figure 13.3.
5. Unit of Thermodynamic Temperature (kelvin). Temperature is not to be confused with heat. Please review the chapter on thermodynamics if you do not know the difference between the two.

The definition of temperature is based on the phase diagram for water (Figure 13.4). The liquid/solid, liquid/vapor, and solid/vapor lines meet at the triple point, where all three phases coexist simultaneously. Although it would seem difficult to attain the triple point experimentally because the pressure and temperature combination must be exact, it is actually achieved rather easily. A glass vial is evacuated and then partially filled with liquid water, leaving a vapor-space above the liquid. The partially full vial is then frozen. As the ice melts, all three phases will coexist: ice, liquid, and vapor.

Because the triple point of water is rather easily obtained, it is ideal for defining a temperature scale. By definition, the triple point of water is assigned the value 273.16 K and absolute zero is assigned the value 0 K. The distance from absolute zero to the triple point of water is divided into 273.16 parts, which define the size of the kelvin unit.

The kelvin unit of thermodynamic temperature is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
An auxiliary temperature scale is sanctioned in which Celsius temperature \( t \) (in °C) is related to Kelvin temperature \( T \) (in K) according to the relation

\[
t = T - T_o
\]

(13-3)

where \( T_o = 273.15 \) K. In the Celsius temperature scale, water freezes at 0°C and it boils at 100°C provided the pressure is 1 atm. For most engineering work, the Celsius temperature scale is more convenient than the Kelvin temperature scale.

An instrument is needed to divide the interval from absolute zero to the triple point of water, and to extend beyond. In practice, the interval is divided using many different types of instruments (e.g., constant-volume gas thermometers, acoustic gas thermometers, spectral and total radiation thermometers, and electronic noise thermometers). The easiest instrument to understand is the constant-volume gas thermometer. At very low pressures, real gases behave as perfect (ideal) gases. The perfect (ideal) gas equation defines the relationship between the pressure \( P \), the volume \( V \), the quantity of gas in moles \( n \), and the temperature \( T \),

\[
P = \frac{nR}{V}T = kT
\]

(13-4)

(13-5)

where \( k \) is the proportionality constant. Thus, pressure is directly proportional to temperature. To illustrate how this relationship could be used, imagine that we perform an experiment in which the pressure in the constant-volume gas thermometer is 0.010000 atm at the triple point of water. If we then reduce the temperature so the pressure in the thermometer becomes 0.0050000 atm, we can calculate the temperature as

\[
k = \frac{P_1}{T_1} = \frac{P_2}{T_2}
\]

(13-6)

\[
T_2 = \frac{P_2}{P_1}T_1 = \frac{0.0050000 \text{ atm}}{0.010000 \text{ atm}} \times 273.16 \text{ K} = 136.58 \text{ K}
\]

(13-7)

It is very difficult to make precise and accurate thermometer measurements. The convenient reference points presented in Table 13.2 were determined by very careful thermometry.

6. Unit of Amount of Substance (mole). In chemistry, the number of molecules is extremely important. For example, the perfect gas equation (Equation 13-4) has the term \( n \), which describes the number of gas molecules in terms of moles. The mole often gives students difficulty, perhaps because of its unusual name. It is a term that has been used since about 1902 and is short for “gram-molecule.”

The mole is the amount of substance that contains as many elementary entities as there are atoms in 0.012 kg of carbon-12. When the mole is used, the elementary entities must be specified and may be atoms, molecules, electrons, other particles, or specified groups of such particles.

We can visualize what a mole means by imagining using some very tiny tweezers to count the number of atoms in 12 grams (0.012 kg) of carbon-12. The number we obtain is called
Avogadro’s number and is equal to $6.0221467 \times 10^{23}$. Just as we use the name dozen to describe the number 12, we give a name to this important number.

7. Unit of Luminous Intensity (candela). A unit for luminous intensity is required to describe the brightness of light. Candle flames or incandescent light bulbs were originally used as standards. The current standard uses a monochromatic (i.e., single-color) light source, typically produced by a laser, and an instrument called a radiometer to measure the amount of heat generated when light is absorbed.

The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of a frequency $540 \times 10^{12}$ cycles per second and that has a radiant intensity in that direction of $(1/683)$ watt per steradian.

Figure 13.5 is a schematic representation of the measuring system for the candela.

13.3.3 SI Derived Units

The base units may be combined into the derived units shown in Table 13.3. Some derived units have been assigned special names (Table 13.4). These derived units with special names may even be combined with other units to form new derived units (Table 13.5).

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**TABLE 13.2**

International Temperature Scale (ITS-90) reference points (Reference)

<table>
<thead>
<tr>
<th>Element</th>
<th>Boiling point (K)</th>
<th>Freezing point (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3$He</td>
<td>3.2</td>
<td>302.9146</td>
</tr>
<tr>
<td>$^4$He</td>
<td>4.2</td>
<td>429.7845</td>
</tr>
<tr>
<td>H$_2$</td>
<td>13.8033</td>
<td>505.078</td>
</tr>
<tr>
<td>H$_2$</td>
<td>20.3</td>
<td>692.677</td>
</tr>
<tr>
<td>Ne</td>
<td>24.5561</td>
<td>933.473</td>
</tr>
<tr>
<td>O$_2$</td>
<td>54.3584</td>
<td>1234.93</td>
</tr>
<tr>
<td>Ar</td>
<td>83.8058</td>
<td>1337.33</td>
</tr>
<tr>
<td>Hg</td>
<td>234.3156</td>
<td>1357.77</td>
</tr>
<tr>
<td>H$_2$O</td>
<td>273.16</td>
<td></td>
</tr>
</tbody>
</table>

Note: Boiling and freezing points measured at $P = 101.325$ kPa

---

**FIGURE 13.5**

Schematic representation of the apparatus to measure light intensity.
### TABLE 13.3
Examples of SI derived units (Reference)

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Name</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>square meter</td>
<td>m²</td>
</tr>
<tr>
<td>Volume</td>
<td>cubic meter</td>
<td>m³</td>
</tr>
<tr>
<td>Speed, velocity</td>
<td>meter per second</td>
<td>m/s</td>
</tr>
<tr>
<td>Acceleration</td>
<td>meter per second squared</td>
<td>m/s²</td>
</tr>
<tr>
<td>Wave number</td>
<td>reciprocal meter</td>
<td>m⁻¹</td>
</tr>
<tr>
<td>Density, mass density</td>
<td>kilogram per cubic meter</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Specific volume</td>
<td>cubic meter per kilogram</td>
<td>m³/kg</td>
</tr>
<tr>
<td>Current density</td>
<td>ampere per square meter</td>
<td>A/m²</td>
</tr>
<tr>
<td>Magnetic field strength</td>
<td>ampere per meter</td>
<td>A/m</td>
</tr>
<tr>
<td>Angular velocity</td>
<td>radian per second</td>
<td>rad/s</td>
</tr>
<tr>
<td>Angular acceleration</td>
<td>radian per second squared</td>
<td>rad/s²</td>
</tr>
<tr>
<td>Concentration (amount of substance)</td>
<td>mole per cubic meter</td>
<td>mol/m³</td>
</tr>
<tr>
<td>Luminance</td>
<td>candela per square meter</td>
<td>cd/m²</td>
</tr>
</tbody>
</table>

### TABLE 13.4
SI derived units with special names (Reference)

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Name</th>
<th>Symbol</th>
<th>Expression in Terms of Other Units</th>
<th>Expression in Terms of SI Base Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>hertz</td>
<td>Hz</td>
<td>s⁻¹</td>
<td></td>
</tr>
<tr>
<td>Force</td>
<td>newton</td>
<td>N</td>
<td>m·kg·s⁻²</td>
<td></td>
</tr>
<tr>
<td>Pressure, stress</td>
<td>pascal</td>
<td>Pa</td>
<td>N/m²</td>
<td>m⁻¹·kg·s⁻²</td>
</tr>
<tr>
<td>Energy, work, heat</td>
<td>joule</td>
<td>J</td>
<td>N·m</td>
<td>m²·kg·s⁻²</td>
</tr>
<tr>
<td>Power, radiant flux</td>
<td>watt</td>
<td>W</td>
<td>J/s</td>
<td>m²·kg·s⁻³</td>
</tr>
<tr>
<td>Electric charge</td>
<td>coulomb</td>
<td>C</td>
<td>s·A</td>
<td></td>
</tr>
<tr>
<td>Electric potential</td>
<td>volt</td>
<td>V</td>
<td>W/A</td>
<td>m²·kg·s⁻³·A⁻¹</td>
</tr>
<tr>
<td>Capacitance</td>
<td>farad</td>
<td>F</td>
<td>C/V</td>
<td>m⁻²·kg⁻¹·s⁴·A²</td>
</tr>
<tr>
<td>Electric resistance</td>
<td>ohm</td>
<td>Ω</td>
<td>V/A</td>
<td>m²·kg⁻¹·s⁻²·A⁻²</td>
</tr>
<tr>
<td>Electric conductance</td>
<td>siemens</td>
<td>S</td>
<td>A/V</td>
<td>m⁻²·kg⁻¹·s³·A²</td>
</tr>
<tr>
<td>Magnetic flux</td>
<td>weber</td>
<td>Wb</td>
<td>V·s</td>
<td>m²·kg·s⁻²·A⁻¹</td>
</tr>
<tr>
<td>Magnetic flux density</td>
<td>tesla</td>
<td>T</td>
<td>Wb/m²</td>
<td>kg·s⁻²·A⁻¹</td>
</tr>
<tr>
<td>Inductance</td>
<td>henry</td>
<td>H</td>
<td>Wb/A</td>
<td>m²·kg·s⁻²·A⁻²</td>
</tr>
<tr>
<td>Celsius temperature</td>
<td>degree Celsius</td>
<td>°C</td>
<td>K</td>
<td></td>
</tr>
<tr>
<td>Luminous flux</td>
<td>lumen</td>
<td>lm</td>
<td>cd·sr</td>
<td></td>
</tr>
<tr>
<td>Illuminance</td>
<td>lux</td>
<td>lx</td>
<td>lm/m²</td>
<td>m⁻²·cd·sr</td>
</tr>
<tr>
<td>Activity (of a radionuclide)</td>
<td>becquerel</td>
<td>Bq</td>
<td>s⁻¹</td>
<td></td>
</tr>
<tr>
<td>Absorbed dose, specific energy imparted</td>
<td>gray</td>
<td>Gy</td>
<td>J/kg</td>
<td>m²·s⁻²</td>
</tr>
<tr>
<td>Dose equivalent</td>
<td>sievert</td>
<td>Sv</td>
<td>J/kg</td>
<td>m²·s⁻²</td>
</tr>
</tbody>
</table>
### TABLE 13.5
Examples of SI derived units expressed by means of several names (Reference)

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Name</th>
<th>Symbol</th>
<th>Expression in Terms of SI base units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic viscosity</td>
<td>pascal second</td>
<td>Pa·s</td>
<td>m⁻¹·kg·s⁻¹</td>
</tr>
<tr>
<td>Moment of force</td>
<td>newton meter</td>
<td>N·m</td>
<td>m²·kg·s⁻²</td>
</tr>
<tr>
<td>Surface tension</td>
<td>newton per meter</td>
<td>N/m</td>
<td>kg·s⁻²</td>
</tr>
<tr>
<td>Heat flux density, irradiance</td>
<td>watt per square meter</td>
<td>W/m²</td>
<td>kg·s⁻³</td>
</tr>
<tr>
<td>Heat capacity, entropy</td>
<td>joule per kelvin</td>
<td>J/K</td>
<td>m²·kg·s⁻²·K⁻¹</td>
</tr>
<tr>
<td>Specific heat capacity, specific entropy</td>
<td>joule per kilogram kelvin</td>
<td>J/(kg·K)</td>
<td>m²·s⁻²·K⁻¹</td>
</tr>
<tr>
<td>Specific energy</td>
<td>joule per kilogram</td>
<td>J/kg</td>
<td>m²·s⁻²</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>watt per meter kelvin</td>
<td>W/(m·K)</td>
<td>m·kg·s⁻³·K⁻¹</td>
</tr>
<tr>
<td>Energy density</td>
<td>joule per cubic meter</td>
<td>J/m³</td>
<td>m⁻¹·kg·s⁻²</td>
</tr>
<tr>
<td>Electric field strength</td>
<td>volt per meter</td>
<td>V/m</td>
<td>m·kg·s⁻³·A⁻¹</td>
</tr>
<tr>
<td>Electric charge density</td>
<td>coulomb per cubic meter</td>
<td>C/m³</td>
<td>m³·s·A</td>
</tr>
<tr>
<td>Electric flux density</td>
<td>coulomb per square meter</td>
<td>C/m²</td>
<td>m⁻²·s·A</td>
</tr>
<tr>
<td>Permittivity</td>
<td>farad per meter</td>
<td>F/m</td>
<td>m⁻³·kg⁻¹·s⁴·A²</td>
</tr>
<tr>
<td>Permeability</td>
<td>henry per meter</td>
<td>H/m</td>
<td>m·kg⁻¹·s⁻²·A⁻²</td>
</tr>
<tr>
<td>Molar energy</td>
<td>joule per mole</td>
<td>J/mol</td>
<td>m²·kg·s⁻²·mol⁻¹</td>
</tr>
<tr>
<td>Molar entropy, molar heat capacity</td>
<td>joule per mole kelvin</td>
<td>J/(mol·K)</td>
<td></td>
</tr>
<tr>
<td>Radiant intensity</td>
<td>watt per steradian</td>
<td>W/sr</td>
<td>m²·kg·s⁻³</td>
</tr>
<tr>
<td>Radiance</td>
<td>watt per square meter steradian</td>
<td>W/(m²·sr)</td>
<td>kg·s⁻³</td>
</tr>
<tr>
<td>Exposure (x and γ rays)</td>
<td>coulomb per kilogram</td>
<td>C/kg</td>
<td>kg⁻¹·s·A</td>
</tr>
<tr>
<td>Absorbed dose rate</td>
<td>gray per second</td>
<td>Gy/s</td>
<td>m²·s⁻³</td>
</tr>
</tbody>
</table>

### 13.4 SI PREFIXES

Because scientists and engineers describe quantities that span many orders of magnitude (e.g., the size of the atomic nucleus to distances between galaxies), SI includes the multipliers listed in Table 13.6. It is generally desirable to use the appropriate multipliers so that the number falls between 0.1 and 1000. (For example, the length 1340 m is best written as 1.34 km.) Exceptions are:

1. In particular applications, a single unit may be customary. For example, engineering mechanical drawings often express all dimensions in millimeters (mm), regardless of how large or small the number is. Clothing dimensions often are expressed in centimeters (cm).
2. When numbers are being compared or listed (as in a table), all numbers should be given with a single prefix.

The use of prefixes eliminates ambiguities associated with significant figures. The number 1340 m could have three or four significant figures, depending on if the last zero
is needed merely to place the decimal point. By using the prefix, it is clear that 1.34 km has three significant figures and 1.340 km has four significant figures.

Although prefixes clearly communicate the size of a number, their use in calculations can lead to disasters. IT IS STRONGLY RECOMMENDED THAT ALL NUMBERS IN CALCULATIONS BE CONVERTED TO SCIENTIFIC NOTATION. Thus, if we want to calculate the distance $d$ that light travels in a given time $t$ (say one millisecond) given that the speed of light $c$ is 299.8 Mm/s, then we must put these numbers into scientific notation:

$$d = ct$$

$$d = (299.8 \times 10^6 \text{ m/s})(1 \times 10^{-3} \text{ s}) = 299.8 \times 10^3 \text{ m}$$

Now that we have the answer, we may wish to communicate it with the appropriate prefix. In this case, the distance could be reported as 299.8 km.

The prefixes that represent 1000 raised to a power are recommended. Thus, the prefixes *hecto*, *deka*, *deci*, and *centi* are generally to be avoided. Some units are so commonly expressed in this way (e.g., centimeter) that their use is accepted.

Use of words such as *billion* and *trillion* are to be avoided in describing multiples of a unit, because the American meaning is different from elsewhere:

<table>
<thead>
<tr>
<th>Number</th>
<th>United States</th>
<th>Britain, Germany, France</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^9$</td>
<td>billion</td>
<td>milliard</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>trillion</td>
<td>billion</td>
</tr>
<tr>
<td>$10^{15}$</td>
<td>quadrillion</td>
<td>—</td>
</tr>
<tr>
<td>$10^{18}$</td>
<td>quintillion</td>
<td>trillion</td>
</tr>
</tbody>
</table>

* Outside the United States, “deca” is used extensively.
† Generally to be avoided.
The Celsius temperature scale was designed to describe temperatures within the range of normal use. Therefore, it is customary not to attach prefixes to the °C symbol. For example, the temperature 5240°C would not be written as 5.24 k°C. For very large (or small) temperatures, it is preferable to use the Kelvin temperature scale.

In the United States, it is customary to express each multiple of \(10^3\) with the symbol “M.” (This notation is derived from the Roman numeral for 1000.) For example, a chemical plant that produces 1,000,000 pounds per year of benzene might be described as a 1 MM lb/year plant. Although this notation is customary, its use should be avoided because of obvious conflicts with SI, in which the prefix “M” means a multiple of \(10^6\).

### 13.5 Customary Units Recognized by SI

Table 13.7 shows some customary units that are not formally a part of SI, but are so commonly used that their meaning is regulated by the General Conference on Weights and Measures. Note that the symbol for hour is “h,” not “hr” as is commonly used. Also, note that although the symbol for liter is either “l” or “L,” the use of the lowercase “l” should be avoided because it is easily confused with the number “1.”

Table 13.8 shows some commonly used units that must be experimentally measured. Table 13.9 shows some customary units that are widely used in particular disciplines, but

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Definition</th>
<th>Measured Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minute of time</td>
<td>min</td>
<td>1 min = 60 s</td>
<td></td>
</tr>
<tr>
<td>Hour</td>
<td>h</td>
<td>1 h = 60 min = 3600 s</td>
<td></td>
</tr>
<tr>
<td>Day</td>
<td>d</td>
<td>1 d = 24 h = 86,400 s</td>
<td></td>
</tr>
<tr>
<td>Degree</td>
<td>°</td>
<td>(1° = (\pi/180)) rad</td>
<td></td>
</tr>
<tr>
<td>Minute of arc</td>
<td>′</td>
<td>(1′ = (1/60°) = (\pi/10,800)) rad</td>
<td></td>
</tr>
<tr>
<td>Second of arc</td>
<td>″</td>
<td>(1″ = (1/60′) = (\pi/648,000)) rad</td>
<td></td>
</tr>
<tr>
<td>Liter</td>
<td>l, L</td>
<td>1 L = 1 dm(^3) = 10(^{-3}) m(^3)</td>
<td></td>
</tr>
<tr>
<td>Tonne, metric ton</td>
<td>t</td>
<td>1 t = 10(^3) kg</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The ( ) indicates the uncertainty of the last two significant digits at \(\pm 1\) standard deviation.
are only temporarily recognized by the General Conference on Weights and Measures. The Conference discourages their introduction into new disciplines.

### 13.6 RULES FOR WRITING SI UNITS (REFERENCE)

1. Regular upright type (not italics) is used. The *symbol* is written in lowercase except if it was derived from a proper name. The first letter of a symbol derived from a proper name is capitalized.

   Example: m is the symbol for meter and is written in lowercase letters
   
   N is the symbol for newton and is written with an uppercase letter because it originates from the proper name Newton

   The symbol for liter (L) is an exception, because it was not derived from a proper name. It is capitalized to avoid confusion with the number “1.”

2. The unit *names* are always written in lowercase letters, even if they are derived from a proper name.

   Example: meter is the name for the unit of length
   
   newton is the name for the unit, whereas Newton is the name of the person

   An exception is when the unit starts a sentence.

   Example: Newton is the SI unit of force.  
   
   newton is the SI unit of force.  

   Correct
   
   Incorrect
3. Unit *symbols* are unaltered in the plural (i.e., do not add an “s” to the end of a symbol).
   
   *Example:* The rod length is 3 m.  
   Correct  
   The rod length is 3 ms.  
   Incorrect  
   
   (Note: The addition of the “s” to the symbol for meter completely changed the meaning to “millisecond.”)

4. Plurals of the unit *names* are made using the rules of English grammar.
   
   *Example:* The rod length is three meters.  
   Correct  
   The rod length is three meter.  
   Incorrect  
   
   The following units are identical in the singular and plural:

<table>
<thead>
<tr>
<th>Singular</th>
<th>Plural</th>
</tr>
</thead>
<tbody>
<tr>
<td>lux</td>
<td>lux</td>
</tr>
<tr>
<td>hertz</td>
<td>hertz</td>
</tr>
<tr>
<td>siemens</td>
<td>siemens</td>
</tr>
</tbody>
</table>

5. Do not use self-styled abbreviations.
   
   *Example:* s, A  
   Correct  
   sec, amp  
   Incorrect  

6. A space is placed between the symbol and the number.
   
   *Example:* 5 m  
   Correct  
   5m  
   Incorrect  
   
   Exceptions are: degrees Celsius (°C), degree (°), minutes (‘), and seconds ("), with which there is no space.

   *Example:* 10°C  
   Correct  
   10 °C  
   Incorrect  

7. There is no period following the symbol except if it occurs at the end of the sentence.
   
   *Example:* It took 5 s for the reaction to occur.  
   Correct  
   It took 5 s. for the reaction to occur.  
   Incorrect  

   The rod is 3 m.  
   Correct  
   The rod is 3 m..  
   Incorrect  

8. When a quantity is expressed as a number and unit, and is used as an adjective, then a hyphen separates the number and unit.
   
   *Example:* The 3-m rod buckled.  
   Correct  
   The 3 m rod buckled.  
   Incorrect  

   The rod is 3 m.  
   Correct  
   The rod is 3-m.  
   Incorrect
9. The product of two or more unit symbols may be indicated with a raised dot or a space.

   Example: N·m or N m

The raised dot is preferred in the United States. Where a raised dot is impossible (e.g., computer printouts), a period may be used instead. An exception is the symbol for watt hour, in which the space or raised dot may be eliminated.

   Wh

Correct

10. The product of two or more unit names is indicated by a space (preferred) or a hyphen.

   Example: newton meter or newton-meter

   Incorrect

In the case of watt hour, the space may be omitted.

   watthour

Correct

11. A solidus (oblique stroke, /), a horizontal line, or negative exponents may be used to express a derived unit formed from others by division.

   Example: m/s or \( \frac{m}{s} \) or \( m \cdot s^{-1} \)

   Incorrect

12. The solidus must not be repeated on the same line unless ambiguity can be avoided by parentheses. In complicated cases, negative exponents or parentheses should be used.

   Example: m/s² or m⋅s⁻²

   m/s/s

   m⋅kg/(s³⋅A) or m⋅kg⋅s⁻³⋅A⁻¹

   Correct

   Incorrect

13. When using the solidus notation, multiple symbols in the denominator must be enclosed in parentheses.

   Example: m⋅kg/(s³⋅A)

   m⋅kg/s³/A

   Correct

   Incorrect

14. For SI unit names that contain a ratio or quotient, use the word per rather than the solidus.

   Example: meters per second

   meters/second

   Correct

   Incorrect

15. Powers of units use the modifier squared or cubed after the unit name.

   Example: meters per second squared

   meters per square second

   Correct

   Incorrect

   Exceptions are when the unit describes area or volume.

   Example: kilograms per cubic meter

   kilograms per meter cubed

   Correct

   Incorrect
16. Symbols and unit names should not be mixed in the same expression.

*Example:* joules per kilogram or J/kg or J·kg⁻¹ Correct

joules per kg or J/kilogram or J·kilogram⁻¹ Incorrect

17. SI prefix symbols are written in regular upright type (no italics). There is no space or hyphen between the prefix and the unit symbol.

*Example:* 5 ms Correct

5 m s Incorrect

5 m-s Incorrect

18. The entire name of the prefix is attached to the unit name. No space or hyphen separates them.

*Example:* five milliseconds Correct

five milli seconds Incorrect

five milli-seconds Incorrect

The final vowel is commonly dropped from the prefix in three cases:

megohm, kilohm, hectare Correct

megaohm, kiloohm, hectoare Incorrect

19. The grouping formed by the prefix symbol attached to the unit symbol constitutes a new inseparable symbol that can be raised to a positive or negative power and that can be combined with other unit symbols to form compound unit symbols.

*Example:* 1 cm³ = (10⁻² m)³ = 10⁻⁶ m³
1 cm⁻¹ = (10⁻² m)⁻¹ = 10² m⁻¹
1 µs⁻¹ = (10⁻⁶ s)⁻¹ = 10⁶ s⁻¹
1 V/cm = (1 V)/(10⁻² m) = 10² V/m

20. Compound prefixes formed by combining two or more SI prefixes are not permitted.

*Example:* 1 mg Correct

1 µkg Incorrect

Note that even though the kilogram is the base SI unit, multiples are still formed from the gram.

21. A prefix must have an attached unit and should never be used alone.

*Example:* 10⁶/m³ Correct

M/m³ Incorrect

22. Modifiers are not to be attached to the units.

*Example:* MW of electricity Correct

MWe Incorrect

V of alternating current Correct

Vac Incorrect

Pa of gage pressure Correct

Pag Incorrect
If space is limited, the modifier may be placed in parentheses. For example, “Pa (gage)” could be replaced for “Pa of gage pressure.”

23. Use only one prefix in compound units. Normally, the modifier is attached to the numerator.

Example: mV/m
mV/mm

Correct
Incorrect

An exception is when the kilogram occurs in the denominator.

Example: MJ/kg
kJ/g

Correct
Incorrect

24. Dimensionless numbers are not required to have the units reported. For example, the refractive index \( n \) is the speed of light in a vacuum \( c_2 \) relative to its speed in another medium, \( c_1 \).

\[
n = \frac{c_2}{c_1}
\]

(13-9)

Water has a refractive index of 1.33. It is not necessary to report the units; the same units are used in the numerator (m/s) and denominator (m/s), so they cancel.

In some cases, it is desirable to report the units of dimensionless numbers to avoid confusion. For example, in a mixture containing species A, B, and C, the mass fraction \( x_A \) expresses the mass of species A \( m_A \) relative to the total mass \( m_T \).

\[
x_A = \frac{m_A}{m_T}
\]

(13-10)

Both the numerator and denominator have units of kilograms, so \( x_A \) is a dimensionless number. However, to be absolutely clear, it is best to report the species with the mass.

Example: The mass fraction was 0.1 kg benzene/kg total. Preferred
The mass fraction was 0.1. Avoid

25. Units such as “parts per thousand” and “parts per million” may be used. However, it is absolutely necessary to explain what the “part” is.

Example: The mass fraction of CO\(_2\) was 3.1 parts per million. Correct
The mole fraction of CO\(_2\) was 3.1 parts per million. Correct
The fraction of CO\(_2\) was 3.1 parts per million. Incorrect

The adjectives “mass” and “mole” are absolutely essential to clarify the meaning.

26. Unit symbols are preferred to unit names.

Example: 15 m
15 meters

Preferred
Avoid

Many writing conventions require that integers from one to ten be written using words rather than numbers. Therefore, if the number is written in words, then the unit name should be used rather than the symbol.

Example: three meters
three m

Correct
Incorrect
The SI System of Units has three types of units: supplementary, base, and derived. The supplementary units relate to geometry and define the radian and the steradian. There are seven base units: meter, kilogram, second, ampere, kelvin, mole, and candela. Each unit is precisely defined using a transportable standard, except for the kilogram, which still has a prototype standard. The base units may be combined into a variety of derived units, some of which are abbreviated with special names (e.g., Pa for N/m²).

Because scientists and engineers deal with wide ranges of magnitude, prefixes are employed as multipliers to increase or decrease the size of the units. Great care must be taken when writing units to prevent miscommunication.

**Nomenclature**

- \(c\) speed of light (m/s)
- \(d\) distance (m)
- \(k\) proportionality constant (atm/K)
- \(m\) mass (kg)
- \(n\) moles (mol) or refractive index (dimensionless)
- \(P\) pressure (atm)
- \(R\) universal gas constant (atm·m³/(mol·K))
- \(T\) temperature (K)
- \(t\) temperature (°C) or time (s)
- \(V\) volume (m³)
- \(x\) mass fraction (dimensionless)
- \(\beta\) solid angle (dimensionless)
- \(\theta\) plane angle (dimensionless)

**Further Readings**


**PROBLEMS**

13.1 Correct the following units to reflect the proper SI rules:

(a) 18.3 Newton
(b) 45.6 n
(c) 29.0 meter
(d) 56.9 meter/sec
(e) five m
(f) 23 m/second
(g) 493°K
(h) 89.6 μ m
(i) 68.5 Kg
(j) 98.4 m/s/s
(k) 10 m’s
(l) Mm/ms

13.2 Use an appropriate prefix so the number ranges from 0.1 to 1000:

(a) \(9.8 \times 10^5\) m
(b) \(9.56 \times 10^{10}\) J
(c) 0.000056 s
(d) 1,984,000 m³
(e) \(35.6 \times 10^{-4}\) g
(f) \(92.4 \times 10^7\) N

13.3 A constant-volume gas thermometer is used to measure thermodynamic temperature. At the triple point of water, its pressure is \(1.00000 \times 10^2\) Pa. What is its pressure at:

(a) triple point of H₂
(b) triple point of Ne
13.4 Two astronomer-geodesists, J. B. J. Delambre (1749–1822) and P. F. A. Méchain (1744–1804), were appointed to determine the length of a “quadrant of meridian,” that is, the length from the north pole to the equator along a great circle passing through the poles. They worked from 1792 to 1799 to complete their task. This length was divided into $10^{-7}$ parts, the original definition of the meter. In 1799, based upon their measurements, two fine scratches were placed on a platinum bar separated by the distance of one meter. When it was later discovered that there were some small surveying errors, the original definition was abandoned in favor of the platinum bar. For many years, this prototype was the standard that defined the meter.

Later, as measurements improved, the length of the quadrant of meridian was determined to be 10,002,288.3 m. What was the fractional error and percentage error of the measurements made by Delambre and Méchain? If they had access to modern high-precision equipment, would the two fine scratches on the platinum bar have been placed farther apart or closer together? How much longer, or shorter, would the meter be (in millimeters)?